M.Sc. - II (Mathematics) (NEP Pattern) Semester-III

03NEPMATH03 - Mathematical Methods

P. Pages : 3 **GUG/S/25/16015**

Time : Three Hours Max. Marks : 80

* 3 6 2 7 *

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) If $c_1 \& c_2$ are constants then prove that show that **8**

i) $F\left[c_1 f_1(x)+c_2 f_2(x)\right]=c_1 F\left[f_1(x)\right]+c_2\ F\left[f_2(x)\right]$

ii) $F_5\left[c_1 f_1(x)+c_2 f_2(x)\right]=c_1 F_5\left[f_1(x)\right]+c_2 F_5\left[f_2(x)\right]$

b) If $F\left[f(x); x \to \xi\right]=F(\xi)$,Then show that **8**

$F\left[f(x)\cos ax;\ x \to \xi\right]=\frac{1}{2}\left[F(\xi+a)+F(\xi-a)\right]$

OR

c) State and prove the convolution theorem for the Fourier transform. **8**

d) Evaluate Fourier transform of H(x + a) – H(x – a). **8**

UNIT – II

2. a) Let $f(x)$ and $f''(x)$ be continuous and $f''(x)$ be sectionally continuous in $0 \le x \le a$. Then prove that **8**

i) $\bar{f}_c\left[f''(x);n\right]=-f'(0)+(-1)^n f'(a)-\frac{n^2\pi^2}{a^2}\bar{f}_c(n)$

ii) $\bar{f}_5\left[f''(x);n\right]=\frac{n\pi}{a}f(0)-(-1)^n\frac{n\pi}{a}f(a)-\frac{n^2\pi^2}{a^2}\bar{f}_5(n)$

b) Solve the three dimensional Laplace Equation $\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0$. **8**

$0 \le x \le \pi,\ 0 \le y \le \pi,\ 0 \le z \le \pi$ with the boundary conditions

$V=V_0$, when $y=\pi; V=0$, when $y=0$ and

$V=0$, when $x=0,\pi$ and $V=0$, when $z=0,\pi$

OR

c) Solve the diffusion equation **8**

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}, \; 0 < x < a, t > 0$$

For its solution satisfying the boundary condition.

$\frac{\partial u(0,t)}{\partial x} = \frac{\partial u(a,t)}{\partial x} = 0, \; t > 0$ And the initial condition $u(x,0) = f(x), \; 0 \le x \le a$
Using proper finite Fourier transform.

d) The transverse displacement of elastic membrane u (x, y, t) satisfies the **8**
PDE $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$, under the boundary conditions u = 0 on the boundary, u = f (x, y), ut = g (x, y) at t = 0. Find the displacement after utilizing finite Fourier transform.

UNIT – III

3. a) State and prove the first and second shifting theorems of Laplace transform. **8**

b) Evaluate $L^{-1}\left[\frac{1}{p(p+1)^3}\right]$ **8**

OR

c) Use complex inversion formula to evaluate inverse Laplace transform of **8**
$\bar{f}(p) = \frac{2p^2 - 4}{(p+1)(p-2)(p-3)}$

d) Evaluate $L^{-1}\left[\frac{p}{\left(p^2+4\right)^3}\right]$ by using convolution Theorem. **8**

UNIT – IV

4. a) State & prove the convolution Theorem of Mellin Transform. **8**

b) If $f(x) = \begin{cases} x^n, & 0 < x < a \\ 0, & x > a \end{cases}$ Find Hankel transform of order n of $f(x)$. **8**

OR

c) Solve the differential equation, **8**

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} = 0, r \geq 0, z \geq 0$$ Satisfying the conditions

(i) $u \to \infty$ as $z \to \infty$ and as $r \to \infty$.

(ii) $u = f(r)$, on $z = 0$ $r \geq 0$.

d) Find Mellin inversion of Γs. **8**

5. a) State and prove the linearity property of Fourier transform. **4**

b) Define finite Fourier cosine and sine transform. **4**

c) Find $L\left[H(t); t \to p\right]$. **4**

d) Evaluate $H_1\left[\frac{e^{-ax}}{x} : \xi\right]$ **4**

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